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THE EFFECTS OF A MINIMUM WAGE IN A DSGE MODEL: AN EXTENSION OF THE BÉNASSY MODEL

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RESUMEN

La discusión sobre salarios mínimos encuentra resultados ambiguos. La teoría predice un efecto negativo para la economía, sin embargo otros estudios encuentran efectos positivos. Este trabajo implementa un salario mínimo en el marco de Bénassy (1995a) en términos nominales y reales. La calibración del modelo apoya las conclusiones tradicionales: un salario mínimo disminuye la producción total que no puede ser compensada por un shock positivo de tecnología. El consumo y la inversión disminuyen si la producción lo hace. Así, la economía en su conjunto se ve perjudicada por un salario mínimo.

Palabras clave: Salarios mínimos, mercado de trabajo, ciclos de negocios
Clasificación JEL: J21, J31, E32

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ABSTRACT

Economic analyses discussing minimum wages find ambiguous results. The theory mostly predicts a negative effect for the economy, whereas other studies even find positive effects. This work implements a minimum wage in the well known framework by Bénassy (1995a) in both nominal and real terms. Calibration of the model supports the traditional findings: A minimum wage lowers the overall output which cannot be compensated by a positive technology shock. Consumption and investment decline if output does. Thus, the economy as a whole is hurt by a minimum wage.
Keywords: Minimum wages, labor market, business cycles
JEL Classification: J21, J31, E32

1. INTRODUCTION

The labor market, as "the clearinghouse for the most important factor of production" (Burda, *et al.*, 2002 p. 2) has independent of its state of development, a huge influence on the existence and dynamic behavior of business cycles.

The wage setting influences the outcome of this major market to a large extent. Even when aspiring a strictly positive analysis, it is difficult to exclude the fact that "[...] the wage is such an important determination of economic welfare, especially for low-income persons" (Boal, *et al.*, 1997 p. 87). As we know, a minimum wage is mostly set to protect these low-income persons.

A minimum wage is "the amount below which the wage rate paid by a firm cannot legally fall" (Barro, 2001 p. 385). Such a minimum wage can be set either by contracting[1] or by an external authority. Either way, a

[1] One can just think of a contract between workers and the firm or one between the union and the firm.

minimum wage may be binding. The effects of a wage which is not found by the optimality conditions of the firm and the household participating in the economy are contentious.

If a minimum wage is set to an amount higher than the market clearing wage, the common opinion in the economic society is that it causes a gap between demand and supply of labor leading to increased involuntary unemployment,[2] a minimum wage which was initially meant to rise the income of the poor, tends to hurt them by destroying their jobs. These two straightforward effects display the ambiguity well known to all of us concerning the wage setting.

In contrast, Card and Krueger (1995) find positive effects of a minimum wage on employment in the respective industry. In their publication, Card and Krueger (1995) look at different case studies for the fast-food industry in various American states for periods before and after the implementation of a minimum wage. The authors find a possibly surprising effect: "Relative to high-wage restaurants, employment increased in restaurants affected by the minimum wage" (Card, *et al.*, 1995 p. 56).

This paper implements a minimum wage in a DSGE framework building upon Bénassy (1995a). To the best of my knowledge, no such model has been published at this point of time. This is surprising, as a DSGE model captures actual economies better than most of its ancestors; not only because it implements different kinds of actors, but also because it tries to show a long time-span and allows for shocks. To implement a minimum wage in such a model seems to be the corollary, as minimum wages have been discussed widely since their invention.[3]

For the analysis hereinafter, the seminal model by Bénassy (1995a) will serve as a benchmark. Considered will be a standard economy, but extended by the assumption of incomplete capital depreciation. The subsequent sections

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[2] A broad definition for unemployment might be: "[...] the number of people who are looking for work but have no job" (Barro, 2001 p. 351).

[3] The first minimum wage in the US has been set in 1938 and was at a rate of $0.25 per hour (Barro, 2001 p. 373).

will implement a minimum wage in the model derived before, in both nominal and real terms. The last section will serve as a conclusion and tries to give suggestions for further research on this field.

2. THE BENCHMARK MODEL

The following section builds on the model by Bénassy (1995a) who tries to "construct a simple and explicitly computable model with optimizing agents in an economy with money and wage contracts" (Bénassy, 1995 p. 303). Previous papers mostly assumed Walrasian market clearing; previous research, on the other hand, has shown that especially the implementation of wage rigidities brings the findings of business cycle models much closer to stylized facts from real economies than it would be possible otherwise. By building and using a benchmark case which uses the Walrasian assumptions, Bénassy (1995a) tries to show the economic mechanisms at work when introducing non-clearing markets.

2.1 THE ECONOMY

Following Bénassy (1995a), there are two goods and two agents in the respective economy. The traded goods are labor at the price W_t and a private good used for consumption and traded at the price P_t. The economy consists of two interacting agents, a firm and a household, and is closed.

The firm faces a Cobb-Douglas production function of the following form:

$$Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha} \tag{1}$$

with Y_t for the output, K_t for capital, N_t for labor, the elasticity of production is α, and a technology process or productivity parameter Z_t. All profits earned by the firm are transferred to the household which leads to zero profits for the firm.

A capital accumulation function from Hercowitz and Sampson (1991) of the following log linearized form will be used in order to have a model which fits reality better than one with complete capital depreciation would do:[4]

$$K_{t+1} = AK_t^{1-\delta} I_t^{\delta} \tag{2}$$

with δ for the rate of depreciation and I_t representing the investment in period t.[5] The assumption of incomplete capital depreciation itself is already an extension of the Bénassy (1995a) model.[6]

Solving the firm's maximization problem, we find the respective optimality conditions, given by

$$\kappa_t = \alpha\delta \frac{Y_t}{I_{t-1}} \tag{3}$$

$$\frac{W_t}{P_t} = (1-\alpha) Z_t K_t^{\alpha} N_t^{-\alpha} \tag{4}$$

with κ_t for the real return on investment.

The agent interacting with the firm is the household. It faces the following lifetime utility function:

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[4] For further details, please see, Hercowitz, *et al.*, 1991 p. 1216 ff.

[5] The price of capital q (meant in the style of Tobin's q which is, following Burda and Wyplosz (2001), defined as the ratio between the current value of the installed capital and the costs of replacement of the installed capital (Burda, *et al.*, 2003 S. 171) is here not equal to zero. This q can be found by taking the derivatives w.r.t. K_t and I_t of the capital accumulation equation. All relevant investment information for the firm are captured in this parameter. In the special case here, one can find for q the following expression: $q = \frac{1-\delta}{\delta}\frac{I_t}{K_t}$.

[6] In Bénassy (1995a), we find the assumption of incomplete capital depreciation only in the Appendix.

$$U = \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) + \theta \frac{M_t}{P_t} + V(\bar{N} - N_t) \right] \tag{5}$$

with an initial endowment of time N_t, a consumption parameter C_t, its real money holdings $\frac{M_t}{P_t}$, and the utility from leisure $V(\bar{N} - N_t)$.

The household maximizes its utility with respect to its budget constraint which is given by

$$C_t + \frac{M_t}{P_t} + I_t = \frac{W_t}{P_t} N_t + \kappa_t (I_{t-1} + q_{t-1} K_{t-1}) + \mu_t \frac{M_{t-1}}{P_t} \tag{6}$$

with μ_t for a stochastic multiplicative shock.

Optimizing the household's behavior leads to the standard intertemporal Euler condition for consumption

$$\frac{I_t}{C_t} = \alpha\beta\delta E_t \left(1 + \frac{I_{t+1}}{C_{t+1}}\right) = \frac{\alpha\beta\delta}{1 - \alpha\beta\delta}. \tag{7}$$

By forward substitution and together with the transversality condition, this rewrites to

$$C_t = (1 - \alpha\beta\delta) Y_t \tag{8}$$

$$I_t = \alpha\beta\delta Y_t \tag{9}$$

which shows that, as $\alpha\beta\delta < 1$, consumption and investment both rise with output. This is an obvious result: The economy as a whole produces more which further leads c.p. to an increased income for the household. This additional income is spent on investment and consumption. How much accounts to either parameter depends on the size of $\alpha\beta\delta < 1$.[7] In the example chosen here, the household spends the major part of its income for consumption and invests only a small fraction.

From the household's first order conditions, the money demand,[8] and the transversality condition, one can show that

$$\frac{M_t}{P_t} = \frac{\theta(1-\alpha\beta\delta)}{1-\beta} Y_t = \phi Y_t \quad with \;\; \phi = \frac{\theta(1-\alpha\beta\delta)}{1-\beta} \tag{10}$$

captures the real money holdings as the money demanded must be equal to the initial money holdings.[9]

It can be shown, by using the labor market's equilibrium conditions, that N_t is constant and equal to N.

$$NV'(\bar{N} - N) = \frac{1-\alpha}{1-\alpha\beta\delta} \tag{11}$$

Finally, the whole system can be expressed by four logarithmic equations which are needed to derive the equation for output being of major interest for all upcoming interpretations. Those equations are:

[7] In this paper, the standard values for calibration will be used. $\alpha = 0.4$ as a common parameter value for the US in macroeconomic accounting; $\beta = 0.99$ taken from (Hansen, 1985 p. 319), and $\delta = 0.025$ taken from (Burda, *et al.*, 2002 p. 12). This leads to $\alpha\beta\delta = 0.0099$.

[8] Which can be derived as $\frac{M_t}{P\_tC\_t} = \theta + \beta E_t\left(\frac{M_{t+1}}{P_{t+1} + C_{t+1}}\right)$.

[9] Which is captured in the fact that $M_t = \mu_t M_{t-1}$ must be fulfilled.

$$y_t = z_t + \alpha k_t + (1-\alpha) n_t \tag{12}$$

$$n_t = n + (m_t - Em_t) \tag{13}$$

$$k_{t+1} = \log(A) + (1-\delta) k_t + \delta i_t \tag{14}$$

$$i_t = \log(\alpha\beta\delta) + y_t. \tag{15}$$

Substitution of equation (15) into equation (14) and lagging leads to

$$k_t = \frac{\log(A) + \delta \log(\alpha\beta\delta)}{1-(1-\delta)} + \frac{\delta y_{t-1}}{1-(1-\delta)L} \tag{16}$$

This and equation (13) can be substituted into equation (12) and after some 1
manipulations, one obtains:

$$y_t = \frac{(1-(1-\delta)L)(z_t + (1-\alpha)(m_t - Em_t))}{1-(1-\delta+\alpha\delta)L} + n + \frac{\alpha \log(A) + \alpha\delta \log(\alpha\beta\delta)}{(1-\alpha)\delta} \tag{17}$$

Equation (10) can be written in logarithmic terms and substituted into equation (4) which yields after some straightforward calculations to

$$w_t = E_{t-1} m_t + \log(1-\alpha) - \log(\phi) - n \tag{18}$$

Finally, using equations (17) and (18), yields to an expression for the price

$$p_t = Em_t - \log(\phi) - n - \frac{(1-(1-\delta)L)(z_t + (1-\alpha)(m_t - Em_t))}{1-(1-\delta+\alpha\delta)L} - \frac{\alpha\log(A) + \alpha\delta\log(\alpha\beta\delta)}{(1-\alpha)\delta}$$

2.2 THE ECONOMY AFTER A TECHNOLOGY SHOCK

It has been mentioned before that the assumption of incomplete capital depreciation is not chosen in most publications.[10] In the first place, the reason for this is that it makes the achievement of a closed form solution much more difficult. But the relaxation of complete depreciation is reasonable when trying to construct a model fitting an actual economy. Nevertheless, incomplete depreciation leads to some differences compared to the traditional model.

As δ is defined to be between zero and one, $0 < \delta < 1$, it is obvious to see that the output rises under incomplete capital depreciation. This result for the specific model above is in line with the findings of King, *et al.* (1988): “[...], when there is a lower depreciation rate, it follows that there is a higher steady state capital stock and a lower output-capital ratio” (King, *et al.*, 1988 p. 218). In DSGE models, it is traditionally assumed that $\delta = 1$. If one assumes now a decline of this variable to $\delta = 0.025$ (for each quarter), implying an annual depreciation rate of 10%, this new relationship occurs. Due to the higher capital stock, the household invests more and the firm produces a higher output.

In the following, it will be looked at the economy’s Impulse-Response Functions. It has been chosen to calibrate the model instead of using an econometric test as the latter might reject or accept the model due to secondary criteria.

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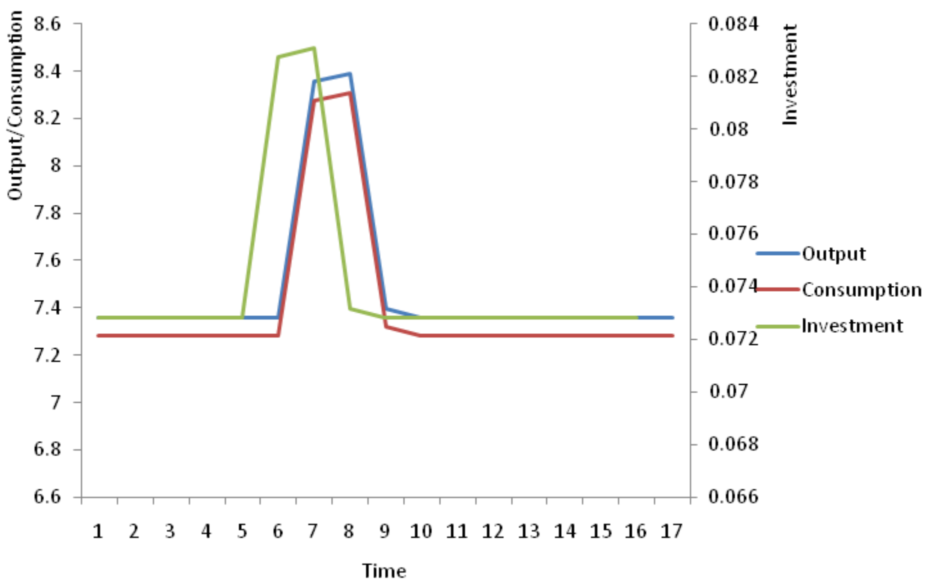
[10] Bénassy (1995a) starts assuming this in his Appendix.

FIGURE 1

The IR Function for Output, Consumption, and Investment under Incomplete Capital Depreciation with a Technology Shock



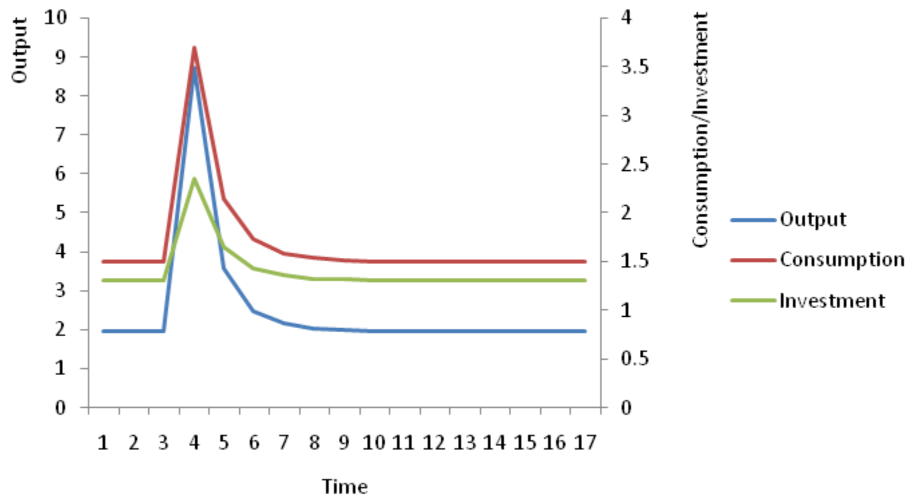
The underlying economy has been hit by a positive technology shock in the seventh quarter of the analyzed time span. Due to the incomplete capital depreciation, the reaction is not as single peaked as it would turn out to be with complete depreciation. The maximum value for output is not reached in the period of the shock, but one quarter later. As there are no other shocks, the initial level is reached again four periods after the shock has arisen.[11]

Looking at the Walrasian regime with complete depreciation, to be seen in Figure 2, it would take output a span of nine quarters to get back to the initial level. Actually, this span would be more than twice as long as it is now with incomplete depreciation.

[11] In the following section will be shown that this is not the case if rigidities occur.

FIGURE 2

Ther IR Function for Output under Complete Capital Depreciation with a Technology Shock in the Walrasian Equilibrium



An additional feature of the incomplete depreciation is that consumption, captured in equation (8), rises if δ declines. The contrary is true for investment, as seen in equation (9). For their Impulse-Response Functions this means, following King, *et al.*, (1988), that "[...] more persistent technology shocks imply that consumption is more responsive and investment is less responsive" (King, *et al.*, 1988 p. 218). This relationship might be hard to see in Figure 1, as it has been chosen to set a second ordinate with a different scale to be able to show all variables in one graph. As there is just a marginal share of income spent on investment, it would lie almost on the abscissa if not proceeding as described.

Independent of the chosen scale, the result is obvious and captured at first sight. All variables react in the same direction, although investment reacts before the other two variables do. In the calibration table, it can be seen that there is already a movement of output and consumption in the period after the shock. This movement is, in relative terms, not as intense as the one for investment. This perfectly makes sense -the depreciation rate has the biggest influence on investment as it affects it directly-. Money is, under the assumption of $\delta < 1$, loosing part of its value in the future periods. As a matter of fact, the household will not spend its income on uncertain investments but uses it for consumption. Additionally, we have assumed that $K_{t+1} = AK_t^{1-\delta} I_t$ which implies that there must be an investment before consumption can take place. This justifies the immediate response of investment.

Of course, there are more variables which would be interesting to analyze at this point. However, we have just built the benchmark case and will now move on to implementing rigidities namely a minimum wage.

3. THE ECONOMY UNDER A MINIMUM WAGE

There are two different ways to implement a minimum wage in an economy like the one derived by Bénassy (1995a) and reviewed in the previous section. The first is to use a nominal minimum wage -with such a wage setting-, it is not possible to comment on the change of the purchasing power for the workers, as inflation is not considered. This will be done in the next section. How to set a real wage will be shown in a later section. In this kind of wage setting, it is of interest how much goods the consumer can actually buy for the nominal wage he gets paid.[12]

3.1 THE ECONOMY UNDER A NOMINAL MINIMUM WAGE

Starting now, it will be assumed that an external authority which might be a union, the government, or a social planner, dictates a wage to be paid on the

[12] The real wage rate can be defined as: "[...] ratio of average hourly earnings of production workers in manufacturing to the consumer price index (CPI)" (Barro, 2001 p. 223).

labor market. This wage, however, is higher than under the fully efficient regime without any disturbances. In the following it will be assumed that $\overline{W_t} > W_t$, with W_t as the nominal minimum wage, in all later periods. As a matter of fact, supply and demand will not be able to reach the efficient clearing point anymore.

Involuntary unemployment may arise as the firm will demand less labor than before due to higher costs and the household, in contrast, wants to supply more hours of work, as it earns a higher wage. Oversupply of work hours arises.

Looking at the firm's problem under the new regime leads to the result that the optimality conditions do not change significantly. Taking the derivative of the firm's profit w.r.t. K_t and N_t leads to

$$\kappa_t = \alpha\delta \frac{Y_t}{I_{t-1}} \tag{20}$$

$$\frac{\overline{W_t}}{P_t}(1-\alpha)Z_t K_t^{\alpha} N_t^{-\alpha} \tag{21}$$

as has been derived in length in the previous section.

The household's Lagrangian does not change either, but its optimality conditions do. Labor is no choice variable anymore, as it is clear that the household will always satisfy the firm's demand for labor with the higher wage. As a matter of fact, there is one FONC less to derive. The remaining ones are:

$$\frac{\partial L}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0 \tag{22}$$

$$\frac{\partial L}{\partial I_t} = -\lambda_t + \beta E_t(\lambda_{t+1}\kappa_{t+1}) = 0 \tag{23}$$

$$\frac{\partial L}{\partial M_t} = \theta \frac{P_t^2}{M_t} + \beta E_t \left[\lambda_{t+1} \frac{\mu_{t+1}}{P_{t+1}} \right] - \lambda_t \frac{1}{P_t} = 0 \tag{24}$$

+ the budget constraint
+ a transversality condition .

Additionally, equations (8), (9), and (10) remain valid.

Considering the firm's optimality conditions, leads to the labor demand function. Straightforward manipulation of equation (21) shows that

$$N_t^D = \left((1-\alpha) Z_t \frac{P_t}{\overline{\overline{W_t}}} \right)^{\frac{1}{\alpha}} K_t \tag{25}$$

The same can be done for equation (20), which leads to the capital equation depending on labor.

$$K_t = \left(\alpha \delta \frac{Z_t}{\kappa_t} \right)^{\frac{1}{1-\alpha}} N_t \tag{26}$$

Substitution of equation (25) into equation (1), results in the following simple expression for output which is not depending on labor anymore:

$$Y_t = Z_t^{\frac{1}{\alpha}} \left((1-\alpha) \frac{P_t}{\overline{\overline{W_t}}} \right)^{\frac{1-\alpha}{\alpha}} K_t \tag{27}$$

To be able to comment on the direct effects of the implementation of a minimum wage, it is reasonable to take the derivative of Y_t w.r.t. $\overline{W_t}$, as it shows the reaction of output even on marginal changes of the wage. This yields to

$$\frac{\partial Y_t}{\partial \overline{W_t}} = -\frac{1-\alpha}{\alpha} Z_t^{\frac{1}{\alpha}} \left((1-\alpha)P_t\right)^{\frac{1-\alpha}{\alpha}} \overline{W_t} K_t < 0 \tag{28}$$

The negative sign in front of this derivative shows an expected outcome: With a nominal minimum wage above the market clearing wage, the output of the economy decreases. This is caused by the disturbance the artificial wage creates; the firm is not able to hire as much labor as it was willing to do in the Walrasian regime, whereas there are workers who face unemployment. If this unemployment is voluntary or involuntary seems not to be important if following Lucas (1978), as "the distinction we are after concerns sources of unemployment, not different types" (Lucas, 1978 p. 354). Here, the source is the minimum wage and that the economy is out of its equilibrium.

Since labor is not the only input factor needed to produce, it is worth looking at the effects on capital as well. For that, one has to take the derivative of equation (26) with respect to $\overline{W_t}$.

$$\frac{\partial K_t}{\partial \overline{W_t}} = \frac{\partial K_t}{\partial N_t}\frac{\partial N_t}{\partial \overline{W_t}} = -\alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)^{\frac{1}{\alpha}}\delta^{\frac{1}{1-\alpha}} Z_t^{\frac{1}{\alpha(1-\alpha)}} P_t^{\frac{1}{\alpha}} \overline{W_t}^{\frac{3\alpha-2}{1-\alpha}} < 0\,. \tag{29}$$

Here as well, the negative effects of a minimum wage are obvious, indicated by the negative sign in front of the derivative. The use of capital declines with increasing $\overline{W_t}$ and with it the demand for labor.

The case of a nominal minimum wage is equal to the wage contract Bénassy (1995a) derives in the analytical sense.[13] None of the equilibrium equations change as Bénassy (1995a) does not assume a finite time horizon for the wage contract. Could the firm take the option into account to be able to renegotiate the wage in the next or any other later period the outcome would differ. As this

[13] For further details, please see Bénassy (1995).

has been ruled out in Bénassy (1995a), the contract wage considerations lead, in fact, to no changes in the variables compared to the case of a nominal minimum wage. Furthermore, it seems to be "reasonable to conjecture that the costs of wage setting leads to the use of long-term contracts" (Fischer, 1977 p. 194) which would be very similar to a nominal minimum wage.

But of course the interpretation changes. By definition, $\overline{W_t} > W_t$ which leads to a decline in all relevant equations compared to those of Bénassy (1995a).[14] Additionally, a minimum wage is different from a contract wage, as the first effects the low-skilled workers only.

3.2 THE EFFECTS OF A NOMINAL MINIMUM WAGE

To be able to comment on the effects caused by a minimum wage, it has been said that output falls due to its introduction. All equations above are, however, still abstract and do not allow a clear picture. To draw such a picture and to make the reasoning more tractable, the model above will be calibrated.

First, it is necessary to find the Walrasian wage, as it is the one paid in $t = 0$. The values presented in Table 1 are either assumed and taken from other studies, or calculated by using the equations derived before.[15]

Choosing these values, equation (18) shows that $w_t = 0.829$. Using equation (25) in logarithmic terms[16] together with equation (10) and assuming a minimum wage higher than w_t,[17] it is straightforward to draw a graph for labor demand and its reaction to the implementation of a minimum wage.

[14] For details, please see Bénassy (1995 p. 308ff).

[15] α common parameter value for the US in macroeconomic accounting; v taken from (Hansen, 1985 p. 323); k_t from equation (9); p_t normalized to 1; y_t and n from the values for the Walrasian outcome derived above; $Em_t = 0.42$ for having adaptive expectations; z_t chosen as before.

[16] $n_t^d = \frac{1}{\alpha}(1-\alpha) + \frac{1}{\alpha}z_t + \frac{1}{\alpha}p_t - \frac{1}{\alpha}\overline{w_t} + k_t$

[17] Here, $\overline{w_t} = 1.2$ has been assumed.

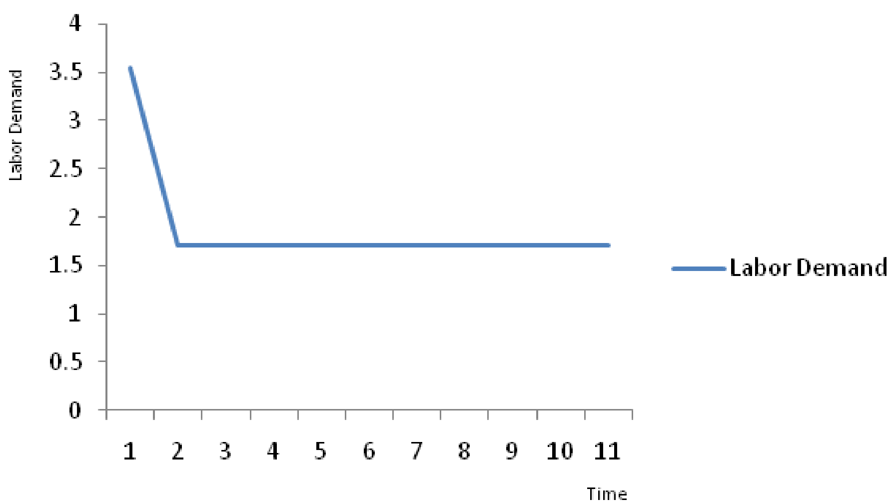
TABLE 1

Parameter Values for Derivation of Employment Changes

Parameter	Value
p_t	1
z_t	1
α	0.4
Em_t	1
ν	2
k_t	0.263
y_t	0.665
n	0.665

FIGURE 3

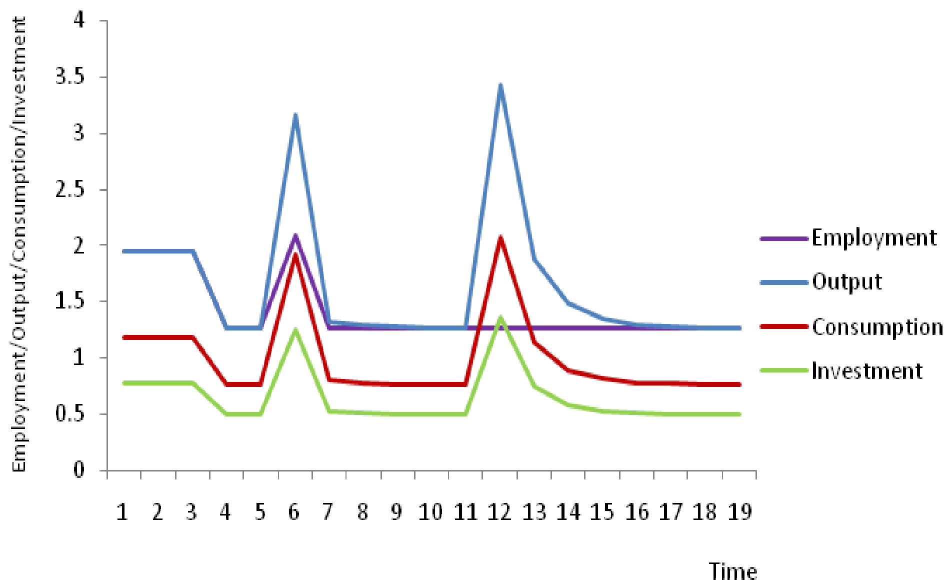
Labor Demand Changes after setting a Minimum Wage



As the curve shows, employment will fall due to the introduction of a minimum wage under the assumption that $\overline{w_t} > w_t$. If there is no return to the competitive wage, the economy will stay at the lower employment level. For the Impulse-Response Function, it is assumed that such a minimum wage occurs in t = 2. However, the question remains what happens after a shock. The economy is hit by a monetary shock (t = 5) and, in addition, a technology shock arises in a later period (t = 11). It must be clear that only output, consumption, and investment react on the technology shock, employment will not, as z_t is not part of the employment equation.

FIGURE 4

The IR Function for Output, Consumption, Employment, and Investment under Incomplete Capital Depreciation with a Technology Shock, a Monetary Shock, and a Nominal Minimum Wage



It is obvious that output reacts stronger to a technology shock than to a monetary shock. However, under the Walrasian regime, the peak would be higher and it would take longer to get back to the initial level (nine periods in the Walrasian regime and six under a wage contract). It is also obvious that the return to the previous level is much smoother after a technology shock, although not smooth enough to reflect the typical hump-shaped reaction.

In comparison to output, employment deviates in relative terms much more after a monetary shock. Due to the structure of equations (8) and (9), investment, consumption, and output deviate, in relative terms, with exactly the same strength. Investment, however, still takes the smallest part of the household's expenditures which as well is due to equations (8) and (9).

The question now arising is whether these results change if we take the change of the purchasing power into account. This will be done in the next section.

3.3 THE ECONOMY UNDER A REAL MINIMUM WAGE

With the assumption of a real minimum wage, the equilibrium conditions for the case of incomplete capital depreciation change to a large extent. The focus here will lie on a real minimum wage, indexed by the missing time subscript *t* and implying that $w_t - p_t < w - p$.

To derive the new equilibrium, we have to look at the labor demand equation which is easily found by considering the household's first order conditions and given by

$$n_t = \frac{1}{\alpha}\log(1-\alpha) + \frac{1}{\alpha}z_t + k_t - \frac{1}{\alpha}(w-p) \tag{30}$$

Therefore, after using equation (14), we find

$$n_t = \frac{1}{1-\alpha}\log(1-\alpha) + \frac{1}{\alpha}z_t + \frac{1}{\alpha}(w-p) + \log(A) + (1-\delta)k_{t-1} + \delta k_t + \delta\log(\alpha\beta\delta) + \delta y_{t-1} \tag{31}$$

From this and after substituting equation (17), one immediately finds the new condition for output.

$$y_t = \frac{1}{\alpha(1-\delta L)} z_t + \frac{1-\alpha}{\alpha(1-\delta)} \log(1-\alpha) + \frac{1-\alpha}{\alpha(1-\delta)} (w-p) + \frac{1}{1-\delta} \log(A) + \frac{\delta}{1-\delta} \log(\alpha\beta\delta) + \underbrace{\frac{1-\delta}{1-\delta L} k_{t-1}}_{\Delta} . \tag{32}$$

The last term of this equation, indicated by Δ, shows the major difference to the case of complete capital depreciation. The equilibrium derived here depends on the capital of the previous period, whereas this would not be the case when assuming complete depreciation. The household has to take this into consideration when making the investment decision in each period. δ shows the rate of capital depreciation, thus, the economic value of capital for the household declines by this rate.

To be able to plot this equation and to make an interpretation easier, it needs to be modified further and Δ must be substituted. One can use equation (14) to do so and lag appropriately.

$$y_t = \frac{1}{\alpha(1-\delta L)} z_t - \frac{1-\delta}{\alpha(1-\delta L)} z_{t-1} + \frac{(1-\alpha)\delta}{\alpha} \left(\log(1-\alpha) - (w-p)\right) + \log(A) + \delta\log(\alpha\beta\delta) \tag{33}$$

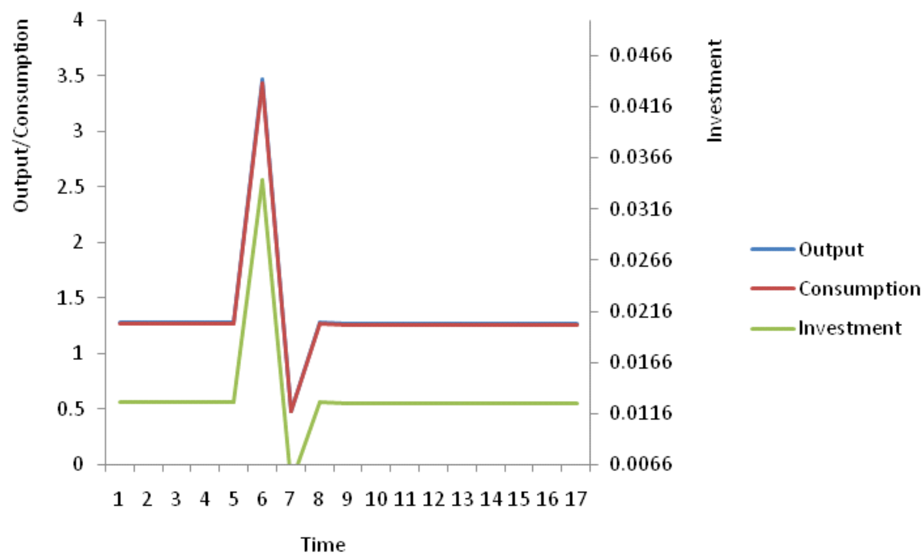
This equation shows the new output for the underlying economy under incomplete capital depreciation and with a real minimum wage.

3.4 THE EFFECTS OF A REAL MINIMUM WAGE

After having derived all necessary equations, one can check again for the Impulse-Response Functions for output, consumption, and investment in the economy.

FIGURE 5

The IR Function for Output, Consumption, and Investment under Incomplete Capital Depreciation with a Technology Shock and a Real Minimum Wage



Here, the technology shock arises in period $t = 5$and the real minimum wage is introduced in $t = 7$. Directly after the technology shock, the output function shows a sharp kink, which has as well been the case in the previous figures. Now, in contrast to the Impulse-Responses shown for the economy without a minimum wage, output declines too much after the shock and cannot reach its initial level again after this strong negative fall. For the reader, this might be difficult to see from the graph but easily from the calibration table which shows an only marginal but still existing deviation from the initial output level.

The real minimum wage implemented here, hinders the economy's output to come back to its exact initial level. Without the real minimum wage, the values would be the same as before the shock has occurred in $t = 8$, meaning three periods after the exogenous driving force has hit the economy. Consumption moves exactly as output, taking even almost the same values. Investment, in contrast takes again a much smaller part from the household's expenditures. But the direction of its movement is as well identical to the one of the other two variables.

The only effect of the real minimum wage, compared to the cases before, is that it lowers the post-shock output level for all future periods. A real minimum wage, set in an economy with incomplete capital depreciation, seems to harm the economy as a whole. With output, investment and consumption decrease as well. This, of course, is only a small decrease, but it cannot be ignored when analyzing the effects of a minimum wage.

4. CONCLUSION

This work aims at extending the standard DSGE world with a minimum wage. This has been done by using the seminal model by Bénassy (1995a) which has been rewritten to allow for rigidities.

The major advantage of using a DSGE model can be seen in the fact that it is robust against the Lucas' critique.[18] Ergo, predictions about the behavior change of agents due to changes in the policy should be captured better in a DSGE framework.

The case of a nominal minimum wage showed that employment reacts the strongest to a monetary shock if there is already a minimum wage implemented. Output, consumption, and investment deviate with the same intensity which is due to the model's structure. After the shocks have died out, the variables of interest cannot get back to their exact pre-shock level. The same is true for a real minimum wage: It hinders the economy to reach its initial output level after a technology shock has occurred. Of course, unemployment appears if

[18] A good overview over the Lucas' critique is given in (Burda, *et al.*, 2003 S. 483f).

the economy is disturbed by a minimum wage. This is the reason why the aggregated utility for all households cannot rise even if some households might be beneficiaries of the minimum wage.

Thus, we have shown that a binding minimum wage has effects in the long-run. It lowers the output level for the economy and with this consumption and investment. Even positive technology shocks cannot compensate for that. For achieving this result, it does not even matter if we use a nominal or real minimum wage, although the effect is larger for the second one.

Lemons (2004) discusses "The Effects of a Minimum Wage on Wages, Employment and Prices" on grounds of a rotating panel data for six Brazilian regions between 1982 and 2000. Especially one of her results is interesting: A 10% higher minimum wage leads to a 0,4% higher overall price in the short-run. In contrast, the same minimum wage enhancement detracts the total employment in the long-run from 0.04%. The effects on the prices seem to be much more significant; the DSGE methodology should be used to find an optimal monetary policy using labor market frictions as an underlying assumption. Particularly in terms of development and inequality for one or the other country, this methodology can give new insights.

Problems of the model used throughout the work might be seen in the exclusion of inflation and the assumption that the household always satisfies the firm's demand for labor. However, it has been used to make a closed form solution easier to achieve. Further research should concentrate on relaxing these assumptions but as well the implementation of a search and matching environment and the possibility of different labor markets might help to better fit reality.[19] This would make the analysis much more complex, but could serve to better show the effects of a minimum wage, as mostly the low-skilled workers are those who are affected by a minimum wage.[20] A good starting point for such an analysis could be to use the framework created by Pissarides (2000)

[19] Different labor markets here refers to low and high skilled labor.

[20] This is due to the fact that a "minimum wage makes the labor of low-productivity workers artificially more expensive" (Barro, 2001 p. 373).

and extended by *e.g.* Moser and Stähler (2009). The challenge will be to transform the Pissarides (2000) framework in a DSGE world.

Of course, a model is only a model if it excludes some parts of reality. But those parts mentioned above seem to be important to achieve explanatory power. Without having explanatory power, suggestions for the political leaders, the unions, the firms, and each regular's table participant are impossible to make. Further research should, no matter if extending the model for inflation or search and matching possibilities, always take into account that a possible introduction of a minimum wage in one of our economies has huge effects on real and nominal variables. And each of the models which might follow in near and further future might be chosen by one of the parties to argue in its sense.

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APPENDIX

1. Derivation of Equation (8) and (9)

$$\lambda_t = \beta E_t(\lambda_{t+1}\kappa_{t+1})$$

$$\frac{1}{C_t} = \beta E_t\left(\frac{1}{C_{t+1}}\alpha\delta\frac{Y_{t+1}}{I_t}\right)$$

$$\frac{I_t}{C_t} = \alpha\beta\delta E_t\left(\frac{I_{t+1} + C_{t+1}}{C_{t+1}}\right)$$

$$\frac{I_t}{C_t} = \alpha\beta\delta + \alpha\beta\delta E_t\left(\frac{I_{t+1}}{C_{t+1}}\right)$$

$$\frac{I_t}{C_t} = \alpha\beta\delta E_t\left(1 + \frac{I_{t+1}}{C_{t+1}}\right)$$

$$\frac{I_t}{C_t} = \frac{\alpha\beta\delta}{1-\alpha\beta\delta}$$

$$I_t = \frac{\alpha\beta\delta}{1-\alpha\beta\delta}C_t$$

$$Y_t = \left(1 + \frac{\alpha\beta\delta}{1-\alpha\beta\delta}\right)C_t$$

$$C_t = (1-\alpha\beta\delta)Y_t$$

$$Y_t = I_t + (1-\alpha\beta\delta)Y_t$$

$$I_t = \alpha\beta\delta Y_t$$

2. Derivation of Equation (10)

$$\frac{M_t}{P_t C_t} = \theta + \beta E_t\left(\frac{M_{t+1}}{P_{t+1} C_{t+1}}\right)$$

$$\frac{M_t}{P_t C_t} = \frac{\theta}{1-\beta}$$

$$\frac{M_t}{P_t(1-\alpha\beta\delta)Y_t} = \frac{\theta}{1-\beta}$$

$$\frac{M_t}{P_t} = \frac{\theta(1-\alpha\beta\delta)}{1-\beta} Y_t$$

$$\frac{M_t}{P_t} = \phi Y_t$$

3. Derivation of Equation (11)

$$\frac{W_t}{P_t} = (1-\alpha)\frac{Y_t}{N_t}$$

$$V'(\overline{N} - N_t) = \lambda_t \frac{W_t}{P_t}$$

$$V'(\overline{N} - N_t) = \frac{1}{C_t}\frac{Y_t}{N_t}$$

$$V'(\overline{N} - N_t) = \frac{1-\alpha}{(1-\alpha\beta\delta)Y_t}\frac{Y_t}{N_t}$$

$$NV'(\overline{N} - N) = \frac{1-\alpha}{1-\alpha\beta\delta}$$

4. Derivation of Equation (16)

$$\mathrm{K}_{\mathrm{t+1}} = \mathrm{AK}_{\mathrm{t}}^{1-\delta}(\alpha\beta\delta\mathrm{Y}_{\mathrm{t}})^{\delta}$$

$$\mathrm{k}_{\mathrm{t+1}} = \log(\mathrm{A}) + \delta\log(\alpha\beta\delta) + (1-\delta)\mathrm{k}_{\mathrm{t}} + \delta\mathrm{y}_{\mathrm{t}}$$

$$\mathrm{k}_{\mathrm{t+1}}(1-(1-\delta)\mathrm{L}) = \log(\mathrm{A}) + \delta\log(\alpha\beta\delta) + \delta\mathrm{y}_{\mathrm{t}}$$

$$k_t = \frac{\log(A) + \delta\log(\alpha\beta\delta)}{1-(1-\delta)} + \frac{\delta y_{t-1}}{1-(1-\delta)L}$$

5. Derivation of Equation (17)

$$y_t = z_t + (1-\alpha)n + (1-\alpha)(m_t - Em_t)$$
$$+ \frac{\log(A) + \delta\log(\alpha\beta\delta)}{1-(1-\delta)} + \frac{\delta y_{t-1}}{1-(1-\delta)L}$$

$$y_t(1-(1-\delta)L)$$
$$= (1-(1-\delta)L)\big(z_t + (1-\alpha)(m_t - Em_t)\big) + (1-\alpha)\big(1-(1-\delta)\big)n$$
$$+\alpha\log(A) + \alpha\delta\log(\alpha\beta\delta) + \alpha\delta y_{t-1}$$

$$y_t(1 - L + \delta L - \alpha\delta L)$$
$$= (1-(1-\delta)L)(z_t + (1-\alpha)(m_t - Em_t) + (1-\alpha)\big(1-(1-\delta)\big)n$$
$$+\alpha\log(A) + \alpha\delta\log(\alpha\beta\delta)$$

$$y_t(1-(1-\delta+\alpha\delta)L)$$

$$=(1-(1-\delta)L)\big(z_t+(1-\alpha)(m_t-Em_t)\big)+(1-\alpha)\big(1-(1-\delta)\big)n$$

$$+\alpha\log(A)+\alpha\delta\log(\alpha\beta\delta)$$

$$y_t=\frac{(1-(1-\delta)L)\big(z_t+(1-\alpha)(m_t-Em_t)\big)}{(1-(1-\delta+\alpha\delta)L)}$$

$$+\frac{(1-\alpha)\big(1-(1-\delta)\big)n}{1-1-\delta+\alpha\delta}+\frac{\alpha\log(A)+\alpha\delta\log(\alpha\beta\delta)}{(1-\alpha)\delta}$$

$$y_t=\frac{(1-(1-\delta)L)\big(z_t+(1-\alpha)(m_t-Em_t)\big)}{1-(1-\delta+\alpha\delta)L}+n$$

$$+\frac{\alpha\log(A)+\alpha\delta\log(\alpha\beta\delta)}{(1-\alpha)\delta}$$

6. Derivation of Equation (19)

$$w_t=\log(1-\alpha)+m_t-\log(\phi)-n$$

$$w_t-p_t=\log(1-\alpha)+y_t-n_t$$

$$w_t-p_t=\log(1-\alpha)-n+n+\frac{(1-(1-\delta)L)\big(z_t+(1-\alpha)(m_t-Em_t)\big)}{1-(1-\delta+\alpha\delta)L}$$

$$+\frac{\alpha\log(A)+\alpha\delta\log(\alpha\beta\delta)}{(1-\alpha)\delta}$$

$$p_t = \log(1-\alpha) + m_t - \log(\phi) - n - \log(1-\alpha)$$

$$-\frac{(1-(1-\delta)L)\big(z_t + (1-\alpha)(m_t - Em_t)\big)}{(1-(1-\delta+\alpha\delta)L)} - \frac{\alpha\log(A) + \alpha\delta\log(\alpha\beta\delta)}{(1-\alpha)\delta}$$

$$p_t = Em_t - \log(\phi) - n$$

$$-\frac{(1-(1-\delta)L)\big(z_t + (1-\alpha)(m_t - Em_t)\big)}{1-(1-\delta+\alpha\delta)L} - \frac{\alpha\log(A) + \alpha\delta\log(\alpha\beta\delta)}{(1-\alpha)\delta}$$

7. Derivation of Equation (31)

$$k_{t+1} = \log(A) + (1-\delta)k_t + \delta\log(\alpha\beta\delta) + \delta y_t$$

$$n_t = \frac{1}{\alpha}\log(1-\alpha) + \frac{1}{\alpha}z_t - \frac{1}{\alpha}(w-p) + k_t$$

$$n_t = \frac{1}{\alpha\log(1-\alpha)} + \frac{1}{\alpha}(w-p) + \log(A)$$

$$+(1-\delta)k_{t-1} + \delta\log(\alpha\beta\delta) + \delta y_{t-1}$$

8. Derivation of Equation (32)

$$y_t = z_t + \alpha k_t + (1-\alpha)n_t$$

$$y_t = z_t + \alpha \log(A) + \alpha(1-\delta)k_{t-1} + \alpha\delta \log(\alpha\beta\delta) + \alpha\delta y_{t-1}$$

$$+\frac{1-\alpha}{\alpha}\log(1-\alpha) + \frac{1-\alpha}{\alpha}z_t - \frac{1-\alpha}{\alpha}(w-p) + (1-\alpha)\log(A)$$

$$+(1-\delta)(1-\alpha)k_{t-1} + \delta(1-\alpha)\log(\alpha\beta\delta) + (1-\alpha)\delta y_{t-1}$$

$$y_t(1-\delta L) = \frac{\alpha z_t + z_t - \alpha z_t}{\alpha} + \frac{1-\alpha}{\alpha}\log(1-\alpha) - \frac{1-\alpha}{\alpha}(w-p)$$

$$+\log(A) + (1-\delta)k_{t-1} + \delta\log(\alpha\beta\delta)$$

$$y_t = \frac{z_t}{\alpha(1-\delta L)} + \frac{1-\alpha}{\alpha(1-\delta)}\log(1-\alpha) - \frac{1-\alpha}{\alpha(1-\delta)}(w-p)$$

$$+\frac{1}{1-\delta}\log(A) + \frac{1-\delta}{1-\delta L}k_{t-1} + \frac{\delta}{1-\delta}\log(\alpha\beta\delta)$$

9. Derivation of Equation (33)

$$y_t = \frac{z_t}{\alpha(1-\delta L)} + \frac{(1-\alpha)\big(\log(1-\alpha) - (w-p)\big)}{\alpha(1-\delta)} + \frac{1}{1-\delta}\log(A)$$

$$+\frac{1-\delta}{1-\delta L}\frac{\log(A) + \delta\log(\alpha\beta\delta)}{1-(1-\delta)} + \frac{1-\delta}{1-\delta L}\frac{\delta y_{t-2}}{1-(1-\delta)L} + \frac{\delta}{1-\delta}\log(\alpha\beta\delta)$$

$$y_t(1-\delta L)\big(1-(1-\delta L)\big)-\delta(1-\delta)y_{t-2}$$

$$=\frac{z_t(1-(1-\delta)L)}{\alpha}+\frac{[(1-\alpha)(\log(1-\alpha)-(w-p))]\delta}{\alpha}$$

$$+\delta\log(A)+(\log(A)+\delta\log(\alpha\beta\delta))(1-\delta)+\delta^2\log(\alpha\beta\delta)$$

$$y_t-\delta y_{t-1}-(1-\delta)y_{t-1}+(1-\delta)\delta y_{t-2}$$

$$=\frac{1-(1-\delta)L}{\alpha}z_t+\frac{(1-\alpha)\delta}{\alpha}(\log(1-\alpha)-(w-p)$$

$$+\log(A)+\delta\log(\alpha\beta\delta))$$

$$y_t(1-\delta L)=\frac{1}{\alpha}z_t-\frac{1-\delta}{\alpha}z_{t-1}+\frac{(1-\alpha)\delta}{\alpha}\big(\log(1-\alpha)-(w-p)\big)$$

$$+\log(A)+\delta\log(\alpha\beta\delta)$$

$$y_t=\frac{1}{\alpha(1-\delta L)}z_t-\frac{1-\delta}{\alpha(1-\delta L)}z_{t-1}+\frac{(1-\alpha)\delta}{\alpha}\big(\log(1-\alpha)-(w-p)\big)$$

$$+\log(A)+\delta\log(\alpha\beta\delta)$$